Handling Uncertainty in Sector Coupled Systems using Dynamic Programming and Model Predictive Control

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Outline

- Model Predictive Control example: Microgrid frequency stabilization
  - Overview
  - Temporal Control Hierarchy
- MPC with Active Learning under Model Uncertainty
  - Model Uncertainties
  - Bayesian recursion
  - Dynamic Programming
- Indirect Control with Active Learning
  - Augmented Objective
  - Temporal Control Hierarchy
- EMPC with Active Learning
  - Objective
  - Temporal Control Hierarchy
- Considered modeling tools
  - System Identification and uncertainty estimation

System model

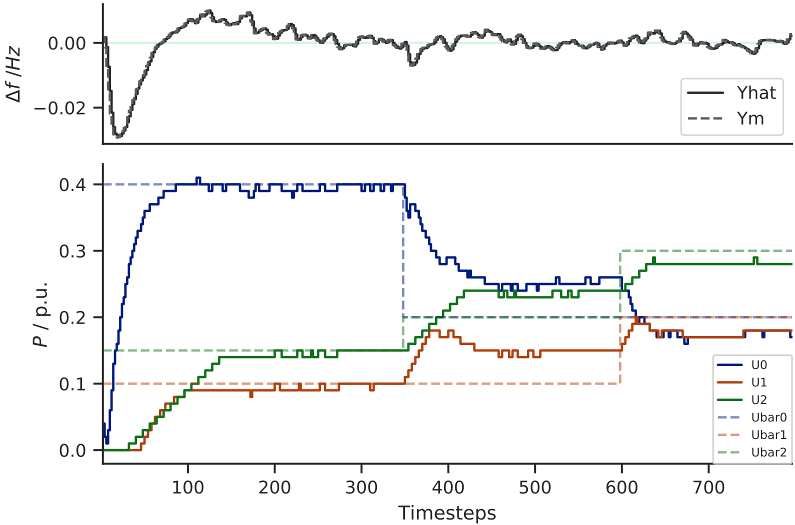
Classical State-Space system:

$$x_{t+1} = Ax_t + Bu_t + Gd_t + w \quad (1)$$
$$y_t = Cx_t + v \quad (2)$$

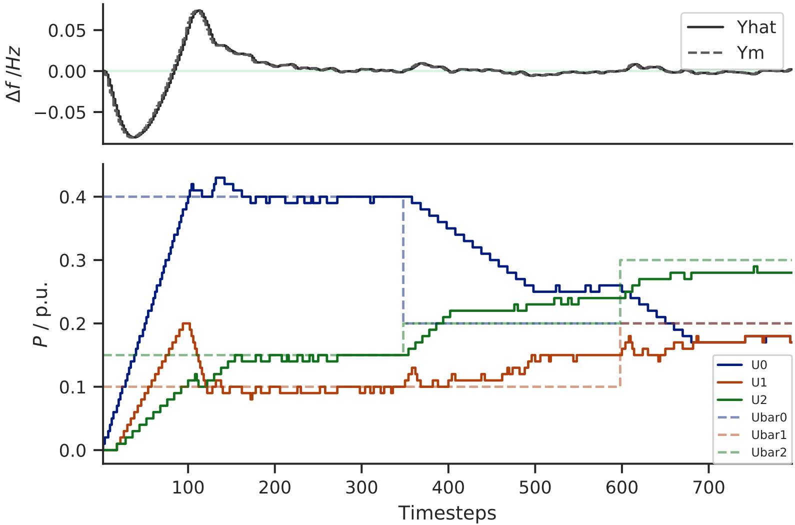
Main state: Swing equation

$$\frac{d}{dt}\Delta f(t) = -\frac{D}{2H}\Delta f(t) + \frac{1}{2H}\Delta P_{\text{mech}}(t) \quad (3)$$

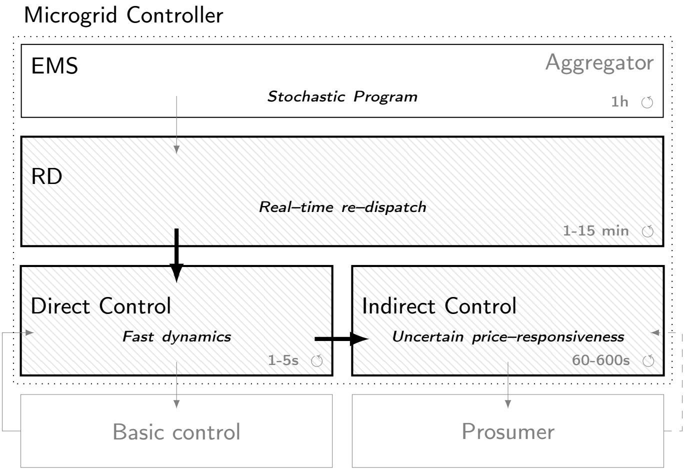
Simulation: High available ramping flexibility



Simulation: Lower available ramping flexibility



Temporal Control Hierarchy with Indirect Control



Exemplary system responses

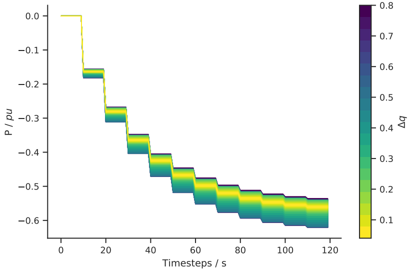


Figure: Small uncertainty response cluster

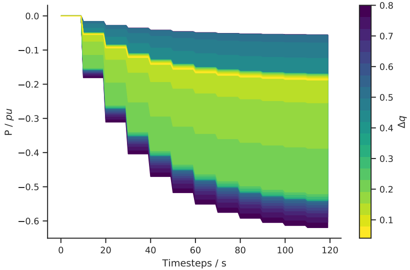
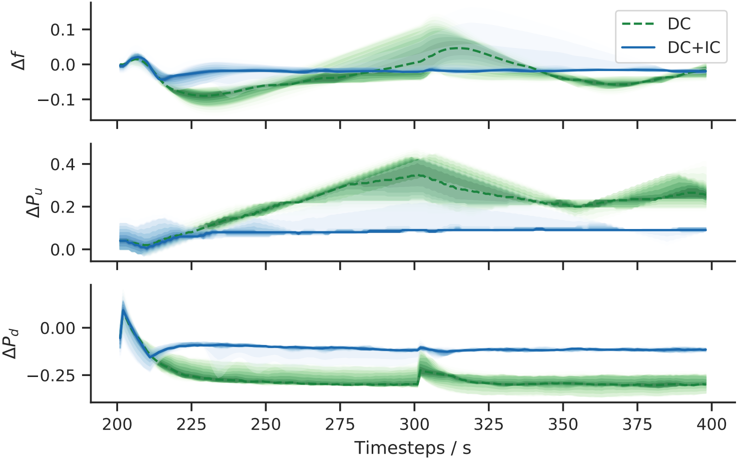


Figure: Large uncertainty response cluster

Simulation: Freq. stab. with uncertain consumption I



DC+IC

Actor (Deterministic) + Prosumer response (Uncertain)

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Actor (Deterministic) + Prosumer response (Uncertain)

Uncertainty should be compensated for post–realization and pro–realization

IC control objective

$$\min_{\{p_{k+j}\}_{j=0}^{N-1}} \Phi_{\mathsf{IC}} = \frac{1}{2}\sum_{j=0}^{N-1} ||\Psi||_Q^2 + ||\Delta p_{k+j}||_R^2 \tag{4}$$

$$\text{s.t.} \quad \hat{x}_{k+1|k} = A\hat{x}_{k|k} + Bp_k \tag{5}$$

$$\hat{x}_{k+1+j|k} = A\hat{x}_{k+j|k} + Bp_{k+j} \tag{6}$$

$$j = 1, 2, \ldots, \mathsf{N} - 1$$

$$\hat{y}_{k+j|k} = C\hat{x}_{k+j|k} \qquad j = 1, 2 \ldots, N \tag{7}$$

$$p_{\mathsf{min}} \leq p_{k+j} \leq p_{\mathsf{max}} \tag{8}$$

Linear/Non–Linear system

Exemplary formulation

$$M_j : \begin{cases} x_{k+1} & = f(x_k, u_k, \theta, w_k) \\ y_k & = h(x_k, \theta_k, v_k) \end{cases} \tag{9}$$

Source of formulation: Heirung et al. 2018

Linear/Non–Linear system

Exemplary formulation

$$M_j : \begin{cases} x_{k+1} & = f(x_k, u_k, \theta, w_k) \\ y_k & = h(x_k, \theta_k, v_k) \end{cases} \tag{9}$$

Source of formulation: Heirung et al. 2018

Sources of uncertainties

- Structural uncertainty
- Parametric uncertainty

Hyperstate propagation (formalized description)

$$\zeta_{k|k-1} = \int p(z_k|z_{k-1}, u_{k-1}) \cdot \zeta_{k-1} dz_{k-1} \tag{10a}$$

$$\zeta_k = \frac{p(y_k|z_k) \cdot \zeta_{k|k-1}}{\int p(y_k|z_k) \cdot \zeta_{k|k-1} dz_k} \tag{10b}$$

Where:

ζ Hyperstate

$z^T = \begin{bmatrix} x & \theta \end{bmatrix}^T$ Augmented state vector

u System input

y System output

DP: General cost function

$$J_k(\zeta_k, \pi_k) = E[\sum_{j=k}^{N-1} l_j(x_j, u_j) + l_N(x_N)] \tag{11}$$

Bellman equation: Recursive problem solution

$$J_k^\star(\zeta_k) = \min_{u_k} \quad E_k[l_k(x_k, u_k) + J_{k+1}^\star(\zeta_{k+1})], \qquad k = 0, 1, \ldots, N-1 \quad (12)$$

IC control objective + Active Learning

$$\min_{\{p_{k+j}\}_{j=0}^{N-1}} \Phi_{\mathsf{IC}} = \frac{1}{2}\sum_{j=0}^{N-1} ||\Psi||_Q^2 + ||\Delta p_{k+j}||_R^2 \tag{13}$$

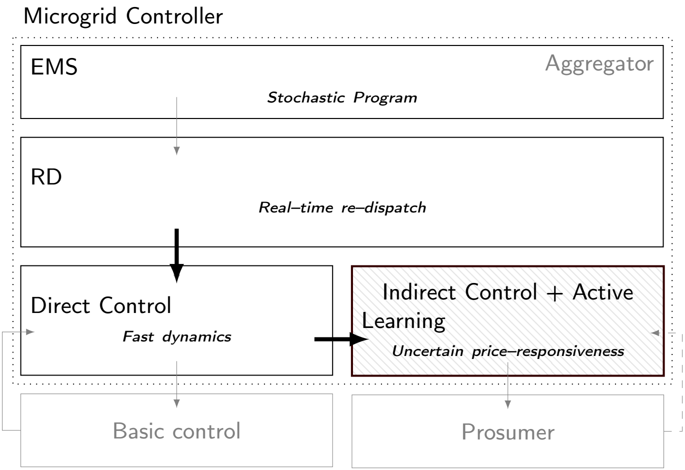
$$\text{s.t.} \quad \hat{x}_{k+1|k} = f(x_k, u_k, \theta_k) \tag{14}$$

$$\hat{x}_{k+1+j|k} = f(x_{k+j}, u_{k+j}, \theta_{k+j}) \tag{15}$$

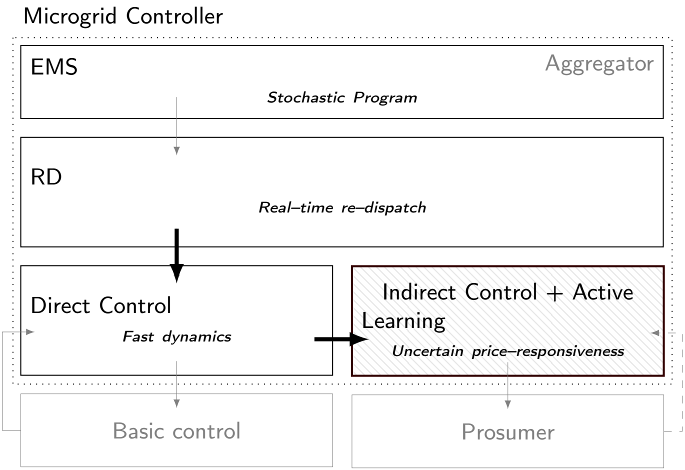
$$j = 1, 2, \ldots, \mathsf{N} - 1$$

$$p_{\mathsf{min}} \leq p_{k+j} \leq p_{\mathsf{max}} \tag{16}$$

Temporal Control Hierarchy



Temporal Control Hierarchy



But: Scope on fast systems

Economic objective function

$$\min_{\{p_{k+j}\}_{j=0}^{N-1}} \Phi_{\text{EMPC}} = \sum_{k\ in \mathcal{N}} p_k u_k + \alpha_v v_k \tag{17}$$

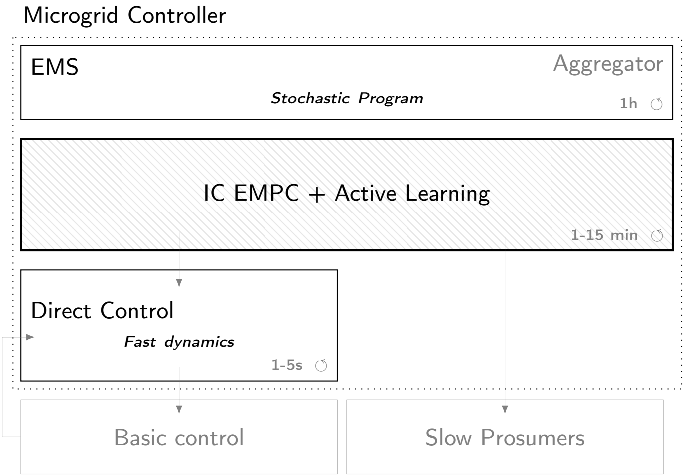
$$\text{s.t.} \quad \hat{x}_{k+1|k} = f(x_k, u_k, \theta_k) \tag{18}$$

$$\hat{x}_{k+1+j|k} = f(x_{k+j}, u_{k+j}, \theta_{k+j}) \tag{19}$$

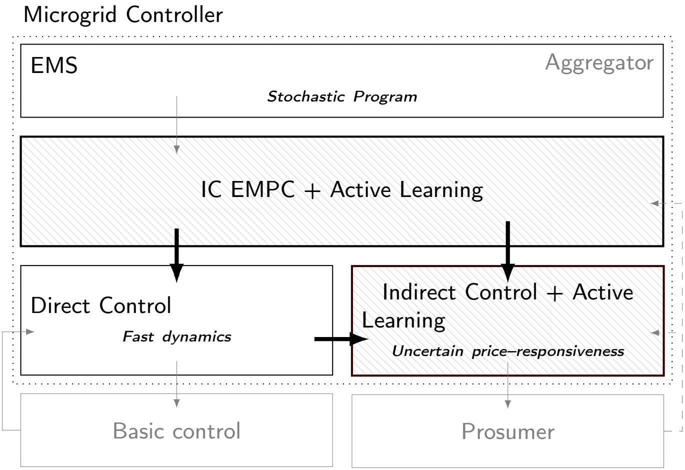
$$j = 1, 2, \ldots, \text{N} - 1$$

$$p_{\text{min}} \leq p_{k+j} \leq p_{\text{max}} \tag{20}$$

Temporal Control Hierarchy: Focus on slow prosumers



Temporal Control Hierarchy: Generalized prosumers



System Identification and uncertainty estimation

- Markov Chain Monte Carlo sampling
  - see e.g. *Stan*
- Classical subspace identification techniques
  - see e.g. Van Overschee and de Moor 1993
- Dynamic Mode Decomposition
  - see e.g. Schmid 2010; Kutz, Fu, and Brunton 2015

Closing notes I

Content:

- Starting point: MPC for aggregated Microgrid operation (Virtual Power Plant)
- Background: MPC with dual effect (Active Learning)
- Goal: Economic MPC considering the dual effect for slow sector coupling

Closing notes II

Thank you for your attention.

uGRIP
microGRId Positioning

CITIES
Centre for IT Intelligent Energy Systems

Figure: This work has been supported by ENERGINET.DK under the project microGRId positioning (uGrip) and the CITIES project.

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P. Van Overschee and B. de Moor. "N4SID: Numerical Algorithms for State Space Subspace System Identification". In: *IFAC Proceedings Volumes*. 12th Triennal Wold Congress of the International Federation of Automatic Control. Volume 5 Associated Technologies and Recent Developments, Sydney, Australia, 18-23 July 26.2, Part 5 (July 1993), pp. 55–58. ISSN: 1474-6670. DOI: 10.1016/S1474-6670(17)48221-8.

Example

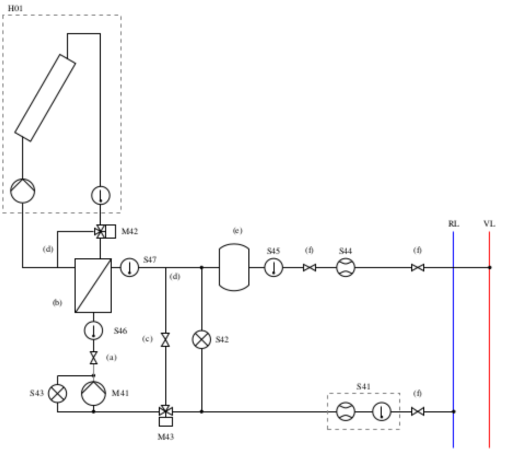

Figure: Solar heat injection station (small scale): Excess heat from solarthermal collectors can be injected in the district heating network.

Reference paper on this regulator formulation

See as reference paper for all aspects on this matter shown below: Banis et al. 2019; Banis et al. 2018. This publication is based on approaches outlined in Pannocchia and Rawlings 2003.

System model

Classical State-Space system:

$$x_{t+1} = Ax_t + Bu_t + Gd_t + w \tag{21}$$

$$y_t = Cx_t + v \tag{22}$$

Main state: Swing equation

$$\frac{d}{dt}\Delta f(t) = -\frac{D}{2H}\Delta f(t) + \frac{1}{2H}\Delta P_{\mathsf{mech}}(t) \tag{23}$$

Objective function

Stabilization problem **T1**

$$J_{\infty,k} = ||\Phi_x(\hat{x}_k - x_{\infty,k}) + \Gamma_u(u_k - u_{\infty,k})||^2 \tag{24}$$

Dynamic Programming problem **T2**

$$J_{\mathsf{DO},k} = ||u_k - u^{\star}_{k-1} + \gamma W_{\Delta u} \Delta u_k||^2 \tag{25}$$

Portfolio constitution **T3**

$$J_{\mathsf{C},k} = (1 - \gamma)\Pi_k \tag{26}$$

Objective function: Overview

$$\min_{u,k} \quad J_{\infty,k} + J_{\mathsf{DO},k} + J_{\mathsf{C},k} \tag{27}$$

$$\text{s.t.} \quad G_k u_k \leq h_k \tag{28}$$

(T1) Residual estimation

Inferring input disturbance[1]

$$\begin{bmatrix} \hat{x}_{k+1|k} \\ \hat{d}_{k+1|k} \end{bmatrix} = \begin{bmatrix} A & B_d \\ 0 & I \end{bmatrix} \begin{bmatrix} \hat{x}_{k|k-1} \\ \hat{d}_{k|k-1} \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u_k + \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} (y_{m,k} - C\hat{x}_{k|k-1} - C_d\hat{d}_{k|k-1}) \quad (29)$$

[1] We optimize over deviations encompassing the positive and negative domain → Only first optimal input required satisfactory, imposing these constraints for the whole sequence $u_{k+N-1|k}$ results in numerical issues.

(T1) Stabilizing gain

Solving for g_∞[2] using least-squares approximation:

$$\overbrace{\begin{bmatrix} A - I & B \\ C & 0 \end{bmatrix}}^{M} \overbrace{\begin{bmatrix} g_{x,\infty} \\ g_{u,\infty} \end{bmatrix}}^{g_\infty} = \begin{bmatrix} B_d \\ 0 \end{bmatrix} \tag{30}$$

$$g_\infty \approx \begin{bmatrix} B_d \\ 0 \end{bmatrix} M^{-1} \tag{31}$$

[2] See Pannocchia and Rawlings 2003; Pannocchia and Rawlings 2001

(T1) Equilibrium point

$$\begin{bmatrix} x_\infty \\ u_\infty \end{bmatrix} = g_\infty \hat{d} \tag{32}$$

(T2) Dynamic programming terms

Ensure offset-free control

$\rightarrow$ Even when constraints are active on parts of the portfolio

(T3) Portfolio constitution

$$\Pi_k = \alpha||u_k - u_{\mathsf{EMS},k}||^2_{W_{\Delta u}} + \beta\left(\, ||\tilde{c}_k u_k||^2 + ||\tilde{c}_{\Delta,k}(u_k - u_{\mathsf{EMS},k})||^2_{W_{\Delta u}} \, \right) \quad \text{where:} \quad \alpha + \beta = 1 \tag{33}$$

Constraints

General

Dynamic reformulation via supervisory system: considering additional system knowledge

$$G_k u_k \leq h_k \tag{34}$$

Particularity: Ramp rate

Only the first optimal input in the sequence required binding[1]

$$\Delta u_{\mathsf{min}} \leq u^{\star}_{k+1|k} - u^{\star}_{k|k} \leq \Delta u_{\mathsf{max}} \tag{35}$$